

Interaction between phantom field and modified Chaplygin gas

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In this letter, we have considered a flat FRW universe. Instead of considering only one candidate for dark energy, we have considered interaction between phantom field and modified Chaplygin gas. It has been shown that the potential of the phantom field increases from a lower value with evolution of the Universe. It has been observed that, the field has an increasing tendency and potential has also an increasing tendency with passage of cosmic time. In the evolution of the universe the crossing of $w = -1$ has been realized by this interacting model.

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In recent observations it has been established that the Universe is made up of roughly two-thirds dark energy that has a negative pressure and can drive the accelerating expansion of the Universe and a plethora of literatures have described these facts [1]. Numerous models have been proposed as candidates of dark energy. Examples of such candidates for dark energy are vacuum energy, quintessence [2], phantom [3] and Chaplygin gas [4]. In order to derive the accelerated expansion of the Universe it is required to have $w < -\frac{1}{3}$. For cosmological constant $w = -1$, for quintessence model $-1 < w < -\frac{1}{3}$ and for k-essence model $w < -1$ or $w > -1$. However, it is physically impossible to cross $w = -1$ [5]. Present observation data constrain the range of the equation of state of dark energy as $-1.38 < w < -0.82$ [6]. However, the equation of state of conventional quintessence models that are based on a scalar field and positive kinetic energy cannot evolve to the regime of $w < -1$. Some authors have investigated a phantom field model which has a negative kinetic energy and can realize $w < -1$ in its evolution [7].

Observations have shown that the current universe is very close to a spatially flat geometry [8]. This is actually a natural result from inflation in the early universe [9]. Consequently, in many recent works the Universe is considered to be spatially flat ($k = 0$) [10 and references therein]. However, recent observations have shown that the spatial curvature of the universe is small. Nevertheless, this result is not sufficient to set conclusions about the topology of the universe. Even in the case of very small curvature, a non-flat model could imply in important consequences on the evolution of the Universe [11].

Neither dark matter nor dark energy has laboratory evidence for its existence directly. Recently, it has been suggested that the change of behavior of the missing energy density might be regulated by the change in the equation of state of the background fluid instead of the form of the potential. This is done by introducing an exotic background fluid, Chaplygin gas, within the framework of Friedman-Robertson-Walker (FRW) cosmology. The Chaplygin gas is characterized by an exotic equation of state $p = -\frac{B}{\rho}$, where B is a positive constant [12]. Role of Chaplygin gas in the accelerated universe has been studied by several authors. The above mentioned equation of state has been generalized to $p = -\frac{B}{\rho^\alpha}$ with $0 \leq \alpha \leq 1$. This is called generalized Chaplygin gas [13]. This equation has been further modified to $p = A\rho - \frac{B}{\rho^\alpha}$ with $0 \leq \alpha \leq 1$. This is called modified Chaplygin gas [14]. This equation of state shows radiation era at one extreme and Λ CDM model at the other extreme.

To obtain a suitable evolution of the Universe an interaction is often assumed such that the decay rate should be proportional to the present value of the Hubble parameter for good fit to the expansion history of the Universe as determined by the Supernovae and CMB data [15]. Guo and Zhang [16] investigated an interaction between matter fluid and phantom field. In the present paper, we have considered an interaction between phantom field and variable Chaplygin gas. The endeavour is to consider a model comprising of two component mixture and to discern how the interaction between two such dark energies influence the evolution and total life time of the universe. To do the same, we have introduced an interaction term which is proportional to the

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Hubble parameter $H = \frac{\dot{a}}{a}$ and the density of the variable modified Chaplygin gas.

The metric for a spatially flat isotropic and homogeneous FRW Universe is given by

$$ds^2 = dt^2 - a^2(t) [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \quad (1)$$

where, $a(t)$ is the scale factor.

The Einstein field equations are

$$3H^2 = \rho_{tot} \quad (2)$$

and

$$6(\dot{H} + H^2) = -(\rho_{tot} + 3p_{tot}) \quad (3)$$

where ρ_{tot} and p_{tot} are the total energy density and isotropic pressure respectively (choosing $8\pi G = c = 1$).

The energy conservation equation is

$$\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0 \quad (4)$$

Considering a two-component model we can write

$$\rho_{tot} = \rho_1 + \rho_2 \quad (5)$$

and

$$p_{tot} = p_1 + p_2 \quad (6)$$

where, ρ_1 , ρ_2 and p_1 , p_2 are energy densities and pressures of phantom field and modified Chaplygin gas respectively.

The stress energy tensor for the phantom field is given by [17]

$$T_{\mu\nu}^{ph} = -\partial_\mu\phi\partial_\nu\phi + g_{\mu\nu} \left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi) \right] \quad (7)$$

Under the assumption that the phantom field is evolving in an isotropic and homogeneous space-time and that ϕ is a function of time alone the energy density ρ_1 and pressure p_1 obtained from $T_{\mu\nu}^{ph}$ are [17]

$$\begin{aligned} \rho_1 &= -\frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p_1 &= -\frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned} \quad (8)$$

Thus, the equation of state parameter is now given by

$$w_1 = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{\frac{1}{2}\dot{\phi}^2 - V(\phi)} \quad (9)$$

From the above expression we find that the equation of state parameter for phantom model is always < -1 .

For modified Chaplygin gas the equation of state is given by

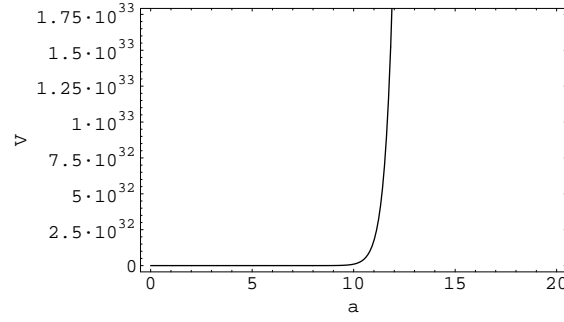


Fig.1

Fig. 1 represents the variation of V against a for $A = 0.1, B = 0.1, C = 0.001, \delta = 0.005, \alpha = 0.5, n = 0.6$.

$$p_2 = A\rho_2 - \frac{B}{\rho_2^\alpha} \quad (10)$$

where, $0 \leq \alpha \leq 1$ and A and B are positive constants.

Now, consider that the phantom field interacting with Chaplygin gas, so the continuity equations for these two fluids can be written through the interaction term Q [18]

$$\begin{aligned} \dot{\rho}_1 + 3H(\rho_1 + p_1) &= Q \\ \dot{\rho}_2 + 3H(\rho_2 + p_2) &= -Q \end{aligned} \quad (11)$$

Note that if $Q > 0$ we have that there exists a transfer of energy from the fluid ρ_2 to the ρ_1 . If $Q = 0$ then we get the non-interacting situation. The nature of the Q term is not clear at all. It may arise in principle from some microscopic mechanisms [18]. For solving these equations different forms for the interaction term Q have been considered in the literature. In the present paper we have taken Q as $Q = 3\delta H\rho_2$ [19], where δ is the coupling constant. Solving the second continuity equation we get

$$\rho_2 = \left(\frac{B}{1 + A + \delta} + \frac{C}{a^{3(1+A+\delta)(1+\alpha)}} \right)^{\frac{1}{1+\alpha}} \quad (12)$$

where, C is the constant of integration. If A and B tend to zero, then $\rho_2 \sim a^{-3(1+\delta)}$. In this case, if we choose $\delta = 0$ then Chaplygin gas behaves like dust matter and $\delta = 1/3$ gives radiation dominated universe. Now for simplicity, we choose $V = n\dot{\phi}^2$, where n is a positive constant. From equation (8), it is observed that n should be greater than $\frac{1}{2}$. Using the above solution for ρ_2 in the first continuity equation, we obtain

$$\dot{\phi}^2 = -\frac{B\delta}{1 + A + \delta} + \frac{6C\delta a^{-3(1+\alpha)(1+A+\delta)}}{(3 - 6n)(1 + \alpha)(1 + A + \delta) - 6} + C_1 a^{\frac{6}{2n-1}} \quad (13)$$

and

$$V = nC_1 a^{\frac{6}{2n-1}} - n\delta \left(\frac{B}{1 + A + \delta} + \frac{Ca^{-3(1+\alpha)(1+A+\delta)}}{1 + \frac{1}{2}(2n-1)(1+\alpha)(1+A+\delta)} \right) \quad (14)$$

In the above expression, V has been expressed as a function of the scale factor a . The variation of V with variation of a is presented in figure 1. It is apparent from the figure 1 that V has a sharp increase from a lower

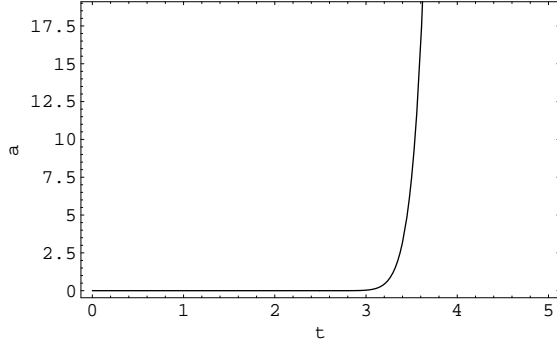


Fig.2

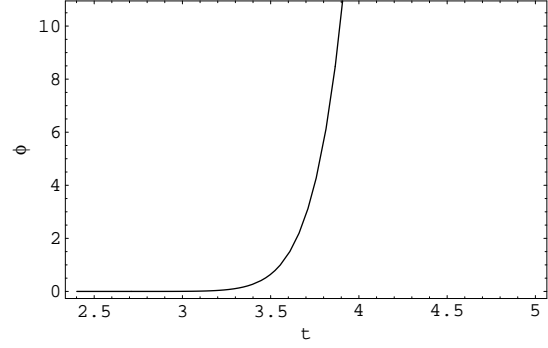


Fig.3

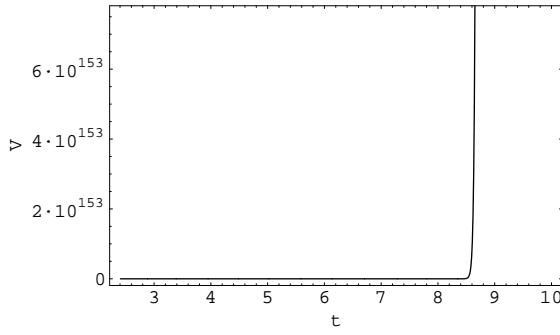


Fig.4

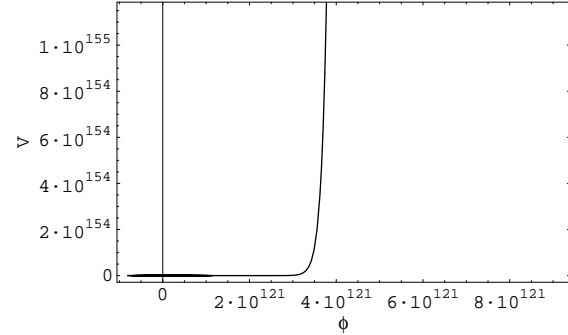


Fig.5

Figs. 2, 3 and 4 represent the variations of a , ϕ and V against t respectively and Fig. 5 represents the variation of V against ϕ with $A = 0.1, B = 0.1, C = 0.001, \delta = 0.005, \alpha = 1, n = 0.6$.

value with increase in a . The variations of a , ϕ and V with variation of t has been presented in figures 2, 3 and 4 respectively. We find that a , ϕ and V gradually increases with increase in t . In figure 5, we have presented the variation of V with ϕ and we see that V gradually increases with increase in ϕ . In figure 6, we have presented the evolution of effective equation of state parameter $w = \frac{p_1 + p_2}{\rho_1 + \rho_2}$ with time t . From the graphical representation we have realized the crossing of $w = -1$ for the interacting dark energy model in evolution of the universe. So this interacting model generates the whole evolution of the universe from quintessence to phantom barrier.

In this letter, we have considered a flat FRW universe. Instead of considering only one candidate for dark energy we have considered phantom field and modified Chaplygin gas. Interaction has been considered between them. Accordingly, an interaction term has been introduced in the conservation equation. An expression for the potential V has been generated under the situation of interaction. From equation (11), it has been found that the energy of Chaplygin gas is getting transferred to phantom field. Subsequently the variation of V has been studied with variation of the scale factor. It has been observed that the potential of the phantom field increases from a lower value with evolution of the Universe. It has been observed that, the field has an increasing tendency and potential has a increasing tendency with passage of cosmic time. Thus, it can be concluded that in the presence of interaction, potential increases and field increases with evolution of the Universe. We have further realized the crossing of $w = -1$ for the interacting dark energy model in evolution of the universe from quintessence to phantom barrier.

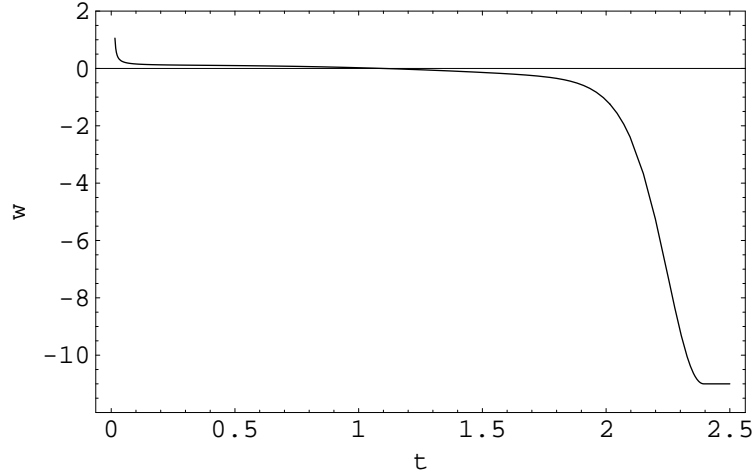


Fig.6

Fig. 6 represents the evolution of w against t for $A = 0.1, B = 0.1, C = 0.001, \delta = 0.005, \alpha = 0.5, n = 0.6$.

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